Weighted Model Counting in FO$^2$ with Cardinality Constraints and Counting Quantifiers

Sagar Malhotra$^1$, Luciano Serafini$^1$

$^1$Fondazione Bruno Kessler, Italy
$^2$University of Trento, Italy

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**Unary and Binary Properties in FO$^2$**

Let us have a FOL language with a unary predicate $\text{k}_i$ and a binary predicate $\text{R}_{xy}$. Then, for any domain constant $c$ exactly one of the following unary property is true:

$\text{Ac} \land \text{Rec} \lor \text{Ac} \land \neg \text{Rec} \lor \neg \text{Ac} \land \text{Rec} \lor \neg \text{Ac} \land \neg \text{Rec} \tag{1}$

For 5 domain elements some examples of unary configurations are given as follows:

- For $\text{Ac} \land \text{Rec}$ Configuration:
- For $\text{Ac} \land \neg \text{Rec}$ Configuration:
- For $\neg \text{Ac} \land \text{Rec}$ Configuration:
- For $\neg \text{Ac} \land \neg \text{Rec}$ Configuration:

In general, for a language with $n$ unary properties over a domain elements, we have $2^n$ ways such that $k_i$ constants realize the $i^{th}$ property. For any pair of domain constants $(c, d)$, exactly one of the following binary properties is true:

$\text{Red} \land \text{Red} \lor \text{Red} \land \neg \text{Red} \lor \neg \text{Red} \land \text{Red} \lor \neg \text{Red} \land \neg \text{Red} \tag{2}$

Given a unary configuration, for each set of domain elements, the following binary configurations are possible:

- Configuration:
- Configuration:
- Configuration:
- Configuration:

In general, for a language with $n$ binary properties given a configuration of unary properties by $\text{R}_{xy}$, then for any pair of unary properties $(c, d)$, we have $2^{\binom{n}{2}}$ possible ways such that $\text{R}_{xy}$ configurations can realize the $j^{th}$ binary property, where

$k(i,j) \in \{k_1, k_2, k_3, k_4\}$

with $k_1 = 1$, $k_2 = 2$, $k_3 = 3$, $k_4 = 4$.

**Existential Quantifiers (Special Case)**

FOMC($\forall x, \forall y, \text{F}(x, y) \land \forall x, y, \neg \text{F}(x, y)$)

<table>
<thead>
<tr>
<th>$\Phi$</th>
<th>$\neg \Phi$</th>
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**Counting Quantifiers (Special Case)**

FOMC($\forall x, \forall y, \text{F}(x, y) \land \forall x, y, \neg \text{F}(x, y)$)

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**Weighed Model Counting**

FOMC can be converted to WPOMC by just adding a multiplicative factor $w(k, \bar{k})$ to every occurrence of $F(k, \bar{k}, (\text{R}_{xy}))$ in any counting formula:

$w(k, \bar{k}) \in E^*$

$w(k, \bar{k})$ is a strictly more expressive weight function than symmetric weight functions.